S.E. Computer & I.T.
CBGS - Sem(3).
App. Maths - 3

20/11/14.

QP Code : 14544

[3 Hours] [Total Marks: 80

N.B. (1) Question no. **1** is **compulsory.**
(2) Attempt any **three** from the remaining.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find the Laplace Transform of sint cos2t cosht. 5
(b) Find the Fourier series expansion of $f(x) = x^2 \ (-\pi, \pi)$ 5
(c) Find the z-transform of $\left(\frac{1}{3}\right)^{|k|}$ 5
(d) Find the directional derivative of $4xz^2+x^2yz$ at (1, -2, -1) in the direction of $2\bar{i}-\bar{j}-2\bar{k}$ 5

2. (a) Find an analytic function f(z) whose real part is $e^x(x\cos y - y\sin y)$ 6
(b) Find inverse Laplace Transform by using convolution theorem $\frac{1}{(s-3)(s+4)^2}$ 6
(c) Prove that $\bar{F}=(6xy^2-2z^3)\bar{i}+(6x^2y+2yz)\bar{j}+(y^2-6z^2x)\bar{k}$ is a conservative field. Find the scalar potential ϕ such that $\nabla\phi=\bar{F}$. Hence find the workdone by $\bar{F}$ in displacing a particle from A(1,0,2) to B(0,1,1) along AB. 8

3. (a) Find the inverse z-transform of $F(z) = \frac{z^3}{(z-3)(z-2)^2}$ 6

(i) $2 < |z| < 3$ (ii) $|z| > 3$

(b) Find the image of the real axis under the transformation $w=\frac{2}{z+i}$ 6
(c) Obtain the Fourier series expansion of 8
$$f(x) = \pi x;\ 0 \le x \le 1$$
$$= \pi(2-x);\ 1 \le x \le 2$$
Here deduce That $\frac{1}{1^2}+\frac{1}{3^2}+.....=\frac{\pi^2}{8}$

4. (a) Find the Laplace Transform of 6
$$f(t) = E;\ 0 \le t \le p/2$$
$$= -E;\ p/2 \le t \le p,\quad f(t+p)=f(t)$$

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(b) Using Grecen's theorem evaluate $\int_c \frac{1}{y}dx+\frac{1}{x}dy$ where c is the boundary of the region bounded by x=1, x=4, y=1, $y=\sqrt{x}$ 6

(c) Find the Fourier integral for $f(x)= 1-x^2, \; 0\leq x\leq 1$
$= 0 \quad x>1$ 8

Heance evaluate $\int_0^\infty \frac{\lambda\cos\lambda-\sin\lambda}{\lambda^3}\cos\left(\frac{\lambda}{2}\right)d\lambda$

5. (a) If $\bar{F}= x^2\bar{i}+(x-y)\bar{j}+(y+z)\bar{k}$ moves a particle from A(1, 0, 1) to B(2, 1, 2) along line AB. Find the workdone. 6
(b) Find the complex form of fourier series f(x) =sinhax $(-\ell,\ell)$ 6
(c) Solve the differential equation using Laplace Transform. 8
$(D^2+2D+5)\, y=e^{-t}\sin t \quad y(0)= 0 \quad y'(0)= 1$

6. (a) If $\int_0^\infty e^{-2t}\,\text{sn}(t+\alpha)\cos(t-\alpha)\, dt=\frac{3}{8}$ find the value of α. 6

(b) Evaluate $\iint_s (y^2z^2\bar{i}+z^2x^2\bar{j}+z^2y^2\bar{k})\,.\,\bar{n}\,ds$ where s is the hemisphere $x^2+y^2+z^2=1$ above xy- plane and bounded by this plane. 6
(c) Find Half range sine series for $f(x)= \ell x-x^2 \; (0,\ell)$ 8

Hence prove that $\frac{1}{1^6}+\frac{1}{3^6}+....=\frac{\pi^6}{960}$
